

## A Sizing Method for a Multi-Robot System

Philippe Rongier  
LIRMM  
161 Rue ADA  
34392 Montpellier  
France  
rongier@lirmm.fr

Philippe Lucidarme  
LIRMM  
161 Rue ADA  
34392 Montpellier  
France  
lucidarm@lirmm.fr

### Abstract

*This paper addresses a fast method for optimizing and sizing a multi-robot system. To demonstrate the performance of such an approach, a complex heterogeneous system is considered here. It is composed of two populations of robots, having different but complementary abilities. It is shown that it is possible to find the optimum criteria to execute the mission and obtain the best probable solution. It is based on the stochastic model of the Markov chains. The method is applied to a generic mission of multi-robot cooperation. The mission has also been simulated several thousand hundred times on a computer to compare the results and validate the model. It is also possible, with this model, to quickly predict the system evolution from different initial states and study the impact of the variation of some parameters on the global result. Some ongoing experimentations are also introduced.*

Keywords: *Multi-robots, Reactive agents, Stochastic model, Autonomous mobile robots, Cooperation.*

### 1 Introduction

Cooperating multiple mobile robots [1] are now seriously envisaged for lots of applications in such fields as defense, planetary exploration [2], agriculture, nuclear plants, underwater exploration, rescue in hostile environments [3]. Distributed solutions using multi-robot systems present many advantages like robustness, resistance to unexpected disturbances [4], fault tolerance thanks to redundancy, self-adaptation and self-organization [5]. In those reactive systems, each agent is equipped with elementary low level functions and ignores both the global goal of the team and the global state of the universe [6]. The study of the emergence of intelligent behavior is generally demonstrated by

thousands of computer simulations which take a long time [7]. Taking into account that most of the tasks have to be completed in an unstructured environment, we used stochastic models like Markov chain for which mathematical properties have been firmly established.

In this paper a fast method to size a multi-robot system made of heterogeneous agents is presented. Each robot is considered as a reactive agent and has simple behaviors [8]. Section 2 describes the mission the system has to accomplish. This is a generic mission fitting many real robotic missions where cooperation between robots when it exists, is expected to improve the global performance of the system. In section 3, the stochastic model is presented and compared with the results obtained with a computer simulator developed to validate the model. The simulator is presented in section 4 and also helps to compute the basic probabilities of the stochastic model. Section 5 is a study for finding the right parameters of the system to obtain the best performance of the system according to different criteria. Then, some results of ongoing experimentations are presented in section 6 before the conclusion.

### 2 The Mission

The mission takes place in an unknown environment where two home bases are randomly located. The global goal is to gather in the bases a maximum of robots randomly scattered. This is a kind of rescue mission where the system is composed of two sets of autonomous robots but:

- The first set gathers robots that have lost their motion abilities due to failures or accidents so they are unable to move by themselves. On the other

hand their vision and location modules are assumed to be still functioning. They can “see” a base when it is close enough in their field of view. Moreover they can emit a help signal that can be detected within a limited range. Once found, they can be carried to a base. They are called P-Robots.

- The second set, the B-Robots, is made of robots which are able to move but which have a destroyed or inoperable vision module. They move randomly, looking for a home base they can detect when they are very close to it “groping around”.

If one B-Robot receives the help signal of a P-Robot entering its call area, it moves towards it, climbs the gradient field of the signal until it reaches the B-Robot. Then the two gathered form a new hybrid robot, BP-Robot, having the both abilities, the vision of the first and the motion skill of the second. The BP-Robots find one of the two home bases more easily because of the longer range of their vision, which helps a lot.

### 3 The Stochastic Model

A twenty states Markov chain is used to model the entire system composed of 3 B-Robots, 3 P-Robots and 2 home bases. It represents all the possible states in which the system could be. Each state is linked with transition probability modeling the way the system could evolve in another state.

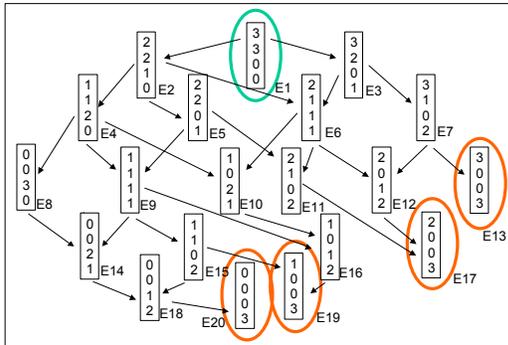


figure 1: Markov chain of the system

The Markov chain is a state representation showing the 20 different states the system could be (figure 1). Each state is a vector :

$$E_n = \begin{bmatrix} N_p \\ N_b \\ N_{bp} \\ N_{hb} \end{bmatrix}$$

table 1: state vector of the system

With  $N_p$  number of P-Robots still in the environment,  $N_b$  number of B-Robots moving randomly,  $N_{bp}$  number of BP-Robots looking for a home base and  $N_{hb}$  the number of robots arrived to the base.

We can notice particular states of the Markov chain:

The initial state (source state):

- E1 where all the robots are in the environment. The P-Robots are calling for help and the B-Robots are moving randomly to find either a home base or a P-Robot call area.

And the four possible ends (sink states):

- E13 is the Case 0, none of the 3 P-Robots have been rescued (the worst case). Only the 3 moving B-Robots have joined the home bases.
- E17 is the Case 1, 1 P-Robot has been carried to the base
- E19 is the Case 2, 2 P-Robots have been rescued.
- E20 is the Case 3, the best case, the one we want to maximize to have the greater probability to rescue the 3 P-Robots.

The others states are the transient states of the system.

From the Markov chain, the stochastic matrix  $S$  is formed. It represents the different possible state transitions and the associated probabilities and provides a complete description of the dynamic operation of the Markov process. To completely define the system at any time, we must compute the probabilities of all the states. This is done by defining the row vector of state probabilities function of  $n$ , the simulation step, as

$$\pi(n) = \left[ \pi_1(n) \quad \pi_2(n) \quad \dots \quad \pi_{20}(n) \right]$$

This vector represents the distribution of the probabilities for the 20 different states of the system. The equation 1

below predicts the future evolution of the modeled system [4].

$$\pi(n) = \pi(0) \cdot S^n$$

**equation 1: probabilistic state transition**

The system is in state E1 at the beginning so

$$\pi(0) = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 1 & 2 & \dots & 20 \end{bmatrix}. \text{ It is also possible and}$$

sometime useful to start from another state. For example if we want to analyze the system when two P-Robots have already been found and one B-Robot is still moving whereas two of the three B-Robots have already joined the home base by themselves (state E15 on figure 1). In this case we have to set the initial probability  $\pi_0$  vector distribution to

$$\pi(0) = \begin{bmatrix} 0 & 0 & \dots & 1 & \dots & 0 \\ 1 & 2 & \dots & 15 & \dots & 20 \end{bmatrix}.$$

Due to the complexity of the theoretical calculus of all the probabilities, the simulation has been used to compute only three basic probabilities. The other transition probabilities linking the different states are linear combinations of these three basic probabilities.

## 4 The Simulator

We have developed a simulator to validate the stochastic model and compute the basic probabilities. Each robot is considered as an agent and each agent is an object in the simulator. The two different behaviors are computed for each of the two kinds of robots as described in the section 2.

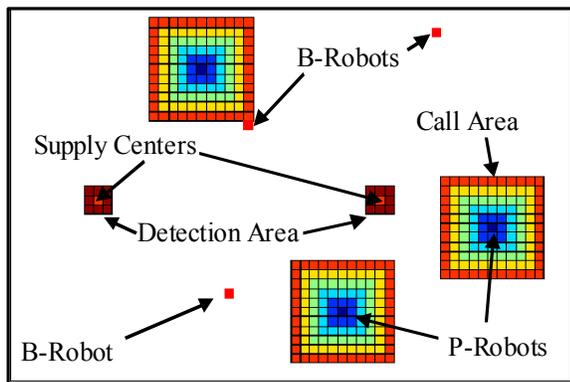


figure 2: snapshot of the simulator

The environment in the simulator is discretized, as a grid of 60x40 squares. Each square is an agent too. It allows to set different parameters for each square of the environment as the value of the signal power which decreases with the distance from the emitting P-Robot. This range is the call area radius of the P-Robot and if a B-Robot enters one of those areas, it receives the help signal and is able to climb the gradient field until reaching the P-Robot to carry it and so forming a new BP-Robot. The BP-Robot is now able to find easily one of the two home bases because it can move and see. The B-Robots move in this environment following a 8-connected random walk. It means that for each step of the simulation, the robot can move in one of the eight neighboring square patches. It has been proved, using this moving method, that all the squares of the environment will be at least visited once in a finite time [9]. The B-Robot can also by chance enter in the detection zone of one home base. If it happens, it moves toward it and stays there.

### 4.1 Setting the Probabilities

The average time a B-Robot spends to find one particular square randomly situated in the environment from a random start location has been computed after many hundred simulations. Then the area radius is increased by one square and so on. We obtain the graph below (figure 3) from a radius of one square to twenty in an environment of 60x40 squares.

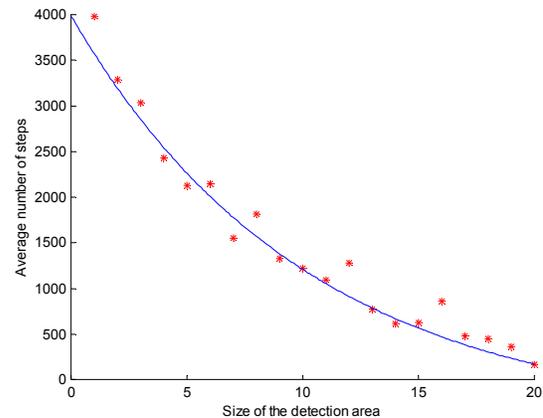


figure 3: average number of steps needed to reach a particular location function of the size of the detecting area

The dots are the mean number of steps as function of the size of the call area radius. The continuous line is an exponential approximation. We can notice on figure 3, the average number of steps a B-Robot needs to reach one home base (size 1) is about 4000 and 2100 to enter a call area (size 5) of a P-Robot. This number of steps falls to 168 for a BP-Robot to see a home base (vision range of 20). With those average number of steps, the basic probabilities are completely defined. The other transition probabilities of the system are linear combinations of these ones. For example the average number of steps needed by one B-Robot to reach one of the two home bases is the basic average time divided by 2, because there are two home bases, that is to say 2000 and so on. The probabilities are deduced from the average number of steps. The Markov chain and the stochastic matrix are thus completely defined .

#### 4.2 Simulation of the System

The system has also been simulated with our simulator to compare and validate the stochastic approach. For 3 B-Robots, 3 P-Robots with a call area size of 5 and a vision range of 15 and with 2 home bases all randomly situated the results are shown below in figure 4:

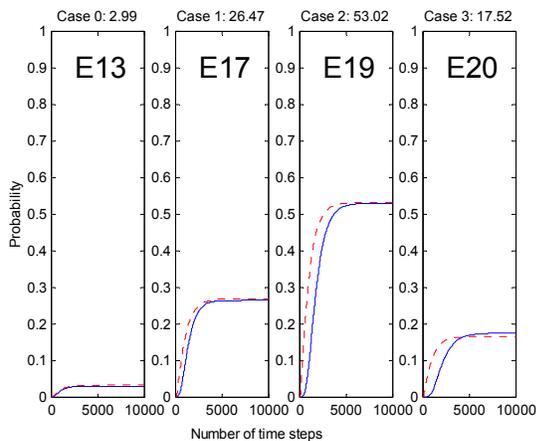


figure 4: comparison between the model and the simulations

The dotted line represents the simulations results, the continuous one, the Markov results. Both give the same results: the evolution of the state probability of the four possible ending states. The model is therefore validated.

## 5 Sizing the Design Parameters

We have already studied the system for 3 B-Robots and 3 P-Robots [11]. The mission has four different final states. The worst case for ending the mission is when none of the P-Robot has been found and carried to a home base, only the 3 B-Robots are in some base (case 0). The best case is obtained when all the 3 P-Robots have been carried by the 3 B-Robots (case 3). This is the full success case. The two intermediate states are when one or two P-Robots have been found (case 1 or case 2). With such a system we obtain (table 2)(figure 4).

End in:	Case 0	Case 1	Case 2	Case 3
Proba:	3%	27%	53%	17%

table 2: probability to end the mission in one of the 4 cases

### 5.1 Focus on the Results of Case 2 and Case 3

The results mean that it is more probable to find one of the two home bases instead of a P-Robot with one B-Robot when 2 P-robots have already been carried to the base. The Markov model results confirm this explanation (figure 5).

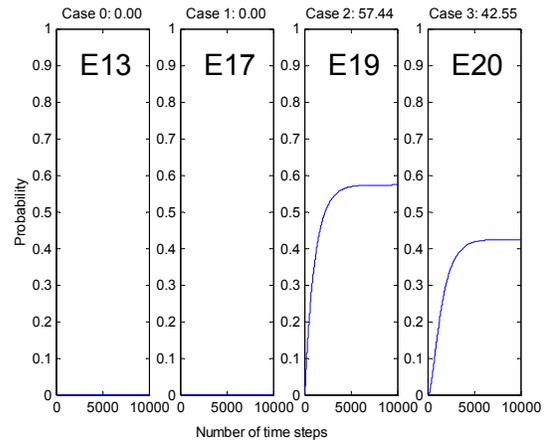


figure 5: probability to end the mission when E15 is the initial state

Those results are obtained when the system starts from the state E15 of the Markov chain (figure 1)(section 3). The only reachable final states are E19 for about 60 % and E20 for about 40 %. That is what happens when the system comes in this state. It

explains the reason why the Case 3 probability of table 2 is lower than the one of the Case 2.

### 5.2 Adding a B-Robot to the System

A new system with 3 P-Robots and this time 4 B-Robots has also been modeled. The Markov chain has then 30 states. The Markov results from the model analysis show the impact on the system when one B-Robot is added to it. Those results are shown in table 3.

End in:	Case 0	Case 1	Case 2	Case 3
Proba:	1%	11%	47%	41%

table 3: probability to end the mission with 4 P-Robots and 3 B-Robots in one of the 4 cases

Adding a moving agent doubles the probability to end the mission in the best case. The percentage reaches now 41%, instead of 20%, to rescue the 3 P-Robots.

### 5.3 Changing the size of the call area

The system made of 3 B-Robots and 3 P-Robots is studied again but this time, by changing some parameters of the robots to see if we can obtain the same kind of results as in table 3. The call area radius of the P-Robots has been increased. The results obtained in table 2 and in table 3 are for a range of 5. With a call range of 7, the results are in table 4.

End in:	Case 0	Case 1	Case 2	Case 3
Proba:	1%	17%	54%	29%

table 4: results for 3 B-Robots and 3 P-Robots with a call range of 7

To obtain these results, the basic probability for a B-Robot to enter a call area according to the average number of steps value of the figure 3 has been changed from size 5 to size 7. The associated values of the stochastic matrix has been changed in the same way.

With a call area size of 9, the results are shown in table 5.

End in:	Case 0	Case 1	Case 2	Case 3
Proba:	1%	10%	50%	39%

table 5: results for 3 B-Robots and 3 P-Robots with a call range of 9

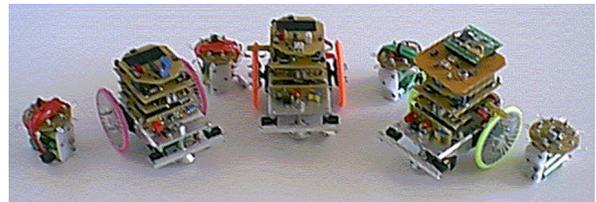
The obtained results in table 5 are the same as those in table 3 when one B-Robot was added to the system.

### 5.4 Conclusion

Increasing the call area range of a P-Robot from 5 to 9 is equivalent to adding a B-Robot to the system. We have about 40% of chance to reach the best case improving the call system of the P-Robots. It could in some cases be worthwhile to equip the robots with a call system with a better range than building another B-Robot depending on the relative costs involved.

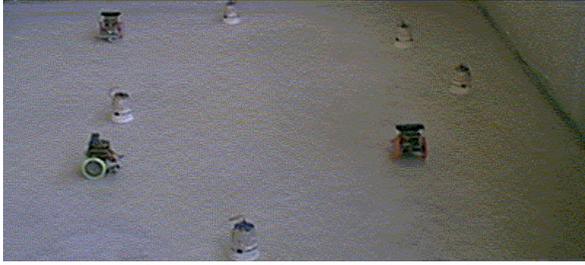
## 6 Real Experimentation

To enhance our study, real mini-robots have been built [12] to compare the real experimental results with those obtained by computer simulations and by the stochastic model [13].



picture 1: 4 active IR beacons and 3 B-Robots

Each B-Robot is equipped with four infra-red receivers. For greater convenience, and ease of use, active infra-red transmitter beacons have been used as P-Robots and home bases (picture 1). The call area radius is proportional to the signal power. The more powerful the signal, larger the call area



picture 2: snapshot of one experimentation

For the experimentation shown on picture 2, the P-Robot signal power has been set five times higher than the signal of the home bases. The radius of the P-Robot call area is equal to 5 and the detection area of the home base is equal to 1. Ongoing experiments confirm the theoretical results.

## 7 Conclusion

The method described in this paper allows to design and size a multi-robots system much more quickly than by long and tedious simulations. Furthermore, it is possible to change quickly some parameters and study the impact on the global result. The obtained results are the different probabilities to end the mission in all the possible different end cases. To validate the method a simulator has been developed. The computer simulation gives the same kind of results but after about 7 hours of simulation whereas the stochastic model results are obtained in less than one minute on a Pentium II 350 MHz. The ongoing experimentations give the results predicted by the stochastic model. The efficiency of this model has been demonstrated by taking as an example a multi heterogeneous robotic system possessing several final states but the method could be applied to other systems. The proposed method allows the multi-robot systems designer to choose the best arrangement to obtain the best result. The robustness of the behavior to unexpected environmental changes and robot failures can be studied and all the possible endings of the system can also be found with their different probabilities.

## References

[1] E. Pagello *et al.*, "Intelligent Multirobot Systems Performing Cooperative Tasks", IEEE Int. Conf.

- on Systems, Man and Cybernetics, Tokyo, pp. 754-760, 1999.
- [2] M.C. Angle, R.A. Brooks, "Small Planetary Rovers", IEEE Int. Workshop on Intelligent Robots and systems, IROS'90, pp. 383-388.
- [3] D. MacKenzie, R. Arkin, Cameron J., "Multiagent Mission Specification and Execution", Autonomous Robots, pp. 1-25, 1997.
- [4] R.A. Books, "A Robust Layered Control System for a Mobile Robot", IEEE Journal of Robotics and Automation, pp. 14-23, 1985.
- [5] J. Ferber, "Multi-Agents System: An introduction to Distributed Intelligence", Addison Wesley Publishing Company, 1999.
- [6] P. Rongier, A. Liégeois, "Analysis and Prediction of the Behavior of one class of Multiple Foraging Robots with the help of Stochastic Petri Nets", IEEE Int. Conf. On Systems, Man and Cybernetics, pp. 143-148, 1999.
- [7] L. Steels, "Towards a Theory of Emergent Functionality", From Animals to Animats, MIT Press, 1991.
- [8] R.C. Arkin, "Behavior-Based Robotics (Intelligent Robots and Autonomous Agents)", the MIT Press, 1998.
- [9] F. Spitzer, "Principles of Random Walk" (2nd ed.), New York, Springer (1976).
- [10] T. Booth, "Sequential Machines and Automata Theory", John Wiley & Sons, Inc., 1967.
- [11] P. Rongier, A. Liégeois, O. Simonin, "Markovian Analysis of a Heterogeneous System: Application to a Cooperation Task for Multiple Consumer Robots", IEEE Int. Conf. On Systems, Man and Cybernetics, pp. 3033-3038, 2000.
- [12] P. Lucidarme, "An autonomous mobile robot: Servobot", <http://51lair.free.fr/menu/robots/servobot/servobot2.htm>, 2000.
- [13] A. Martinoli, A. J. Ijspeert, F. Mondada, "Understanding Collective Aggregation Mechanisms: From Probabilistic Modelling to Experiments with Real Robots", Robotics and Autonomous System, pp. 51-63, 1999.