Internal report : B2P2 modeling and balance criterion computation

Jean-Luc Paillat (jlpaillat@gmail.com), Philippe Lucidarme (philippe.lucidarme@gmail.com) and Laurent Hardouin (laurent.hardouin@istia.univ-angers.fr)

A topically problem with VGTVs is to control the balance. Indeed, the control of this CoG can be a real asset to overcome obstacles and a model of the robot is essential. In this section, two model are used to compute two balance criterion a static one (CoG with geometric model) and a dynamic one (ZMP with the dynamic model).

1 Geometric model

The geometric model is used to define the robot's relative position in a general frame. Thus, it is possible to formulate the CoG in terms of the elements and position of the UGV (the tracks' weight is negligible in regard to the robot's weight). First the robot shape has to be decomposed as it is shown on Fig. 2. Joints 1 to 6 describe the position and the orientation of the robot in the environment. Joints 7 and 8 represent the two actuated joints.

1.1 Denavit & Hartenberg description

From the Fig. 2, the Denavit & Hartenberg (DH) formulation allows the computation of several parameters (table 1) which are used to compute transport matrix in order to formulate the coordinates of a point in any frame of the model described by the vector q of the 8 joints variables :

$$q = [q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8]^T$$

where q_i represents the articular value of the i_{th} joint. For practical purposes the position (q_1, q_2 and q_3) is computed thanks to a GPS, the orientation (q_4) by using a compass, the ground shape (q_5 and q_6) by using an inclination sensor and the two last parameters (q_7 and q_8) are given by the motors 3 and 4 encoders values.

Thanks to these parameters, it is possible to formulate the position of the CoG of each segment in the frame R_0 whatever the position of the different elements of the robot.

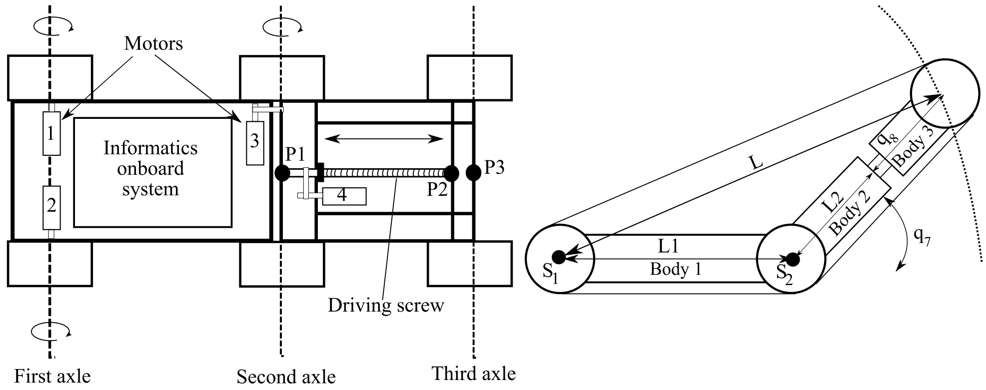

Figure 1: Overview of the B2P2 mechanical structure.

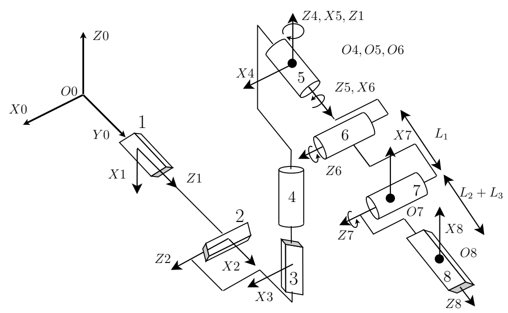

Figure 2: B2P2's geometric model. The robot is decomposed into three segments. The first is situated between joints 6 and 7, the second starts at the joint 7 and the third at the joint 8. L_1, L_2 and L_3 represent respectively the length of the segments 1, 2 and 3.

j	σ_j	α_j	d_j	θ_j	r_j
1	1	$\frac{-\pi}{2}$	0	$\frac{\pi}{2}$	q_1
2	1	$\frac{\pi}{2}$	0	$\frac{\pi}{2}$	q_2
3	1	$\frac{-\pi}{2}$	0	$\frac{-\pi}{2}$	q_3
4	0	0	0	q_4	0
5	0	$\frac{-\pi}{2}$	0	$q_5 - \frac{\pi}{2}$	0
6	0	$\frac{-\pi}{2}$	0	$q_6 - \frac{\pi}{2}$	0
7	0	0	L_1	$q_7 + \frac{\pi}{2}$	0
8	1	$\frac{\pi}{2}$	0	0	$L_2 + q_8$

Table 1: Parameters of Denavit-Hartenberg

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_{R_0} = \frac{m_1 T_6 \begin{bmatrix} z_1 \\ x_1 \\ y_1 \\ 1 \end{bmatrix}_{R_6} + m_2 T_7 \begin{bmatrix} z_2 \\ -y_2 \\ x_2 \\ 1 \end{bmatrix}_{R_7} + m_3 T_8 \begin{bmatrix} -y_3 \\ z_3 \\ x_3 \\ 1 \end{bmatrix}_{R_8}}{m_1 + m_2 + m_3} \tag{1}$$

where x_i, y_i and z_i are the coordinates of the CoG of the i^{th} element of the robot, T_j represents the transport matrix from R_j to R_0, and m_i represents the weight of the i^{th} element of the robot.

2 Dynamic model

This section deals with the dynamic model of the robot which is based on the geometric model (Fig. 2) detailed above. According to this model, the robot motion in a 3D frame (R_0) is described by the vector q of the 8 joints variables :

$$q = [q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8]^T$$

The dynamic model of a mechanical system establishes a relation between the effort applied on the system and its coordinates, generalized speeds and accelerations ([1] and [2]). In this section, the following notations are used :

- j describes the joints from 1 to 8,
- i describes the segments from 1 to 3 (referenced on Fig 1),
- n and m describes indexes from 1 to 8.

2.1 The Dynamic equations

The general dynamic equations of a mechanical system is :

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = Q_j + T_j \tag{2}$$

- L is the Lagrangien of the system. It is composed of rigid segments, so there is no potential energy. Although the Lagrangien corresponds to the kinetic energy.
- q_j is the j^{th} joint variable of the system.
- Q_j is the gravity's torque applied to the j^{th} joint of the system.
- T_j is the external force's torque applied to the j^{th} joint of the system.

The kinetic energy is given by :

$$K = \sum_{i=1}^{n} \frac{1}{2} m_i v_i^T v_i + \frac{1}{2} w_i^T I_i w_i. \tag{3}$$

- m_i is the mass of the i^{th} element of the model,
- v_i is the linear speed of the i^{th} element's center of gravity,
- w_i is the angular speed of the i^{th} element's center of gravity,
- I_i is the matrix of inertia of the i^{th} element of the system.

In order to have homogeneous equations, w_i is defined in the same frame as I_i ; it allows to formulate v_i and w_i according to q :

$$v_i = J_{v_i}(q)\dot{q} \tag{4}$$

$$w_i = R_{0j}^T J_{w_i}(q)\dot{q} \tag{5}$$

where J_{v_i} and J_{w_i} are two matrices and R_{0j} is the transport matrix between the frame R_0 and the frame j linked to the segment i.

The kinetic energy formula is :

$$K = \frac{1}{2}\dot{q}^T \sum_i [m_i J_{v_i}(q)^T J_{v_i}(q) + J_{w_i}^T(q) R_{0j} I_i R_{0j}^T J_{w_i}(q)]\dot{q} \tag{6}$$

which can be rewritten as :

$$K = \frac{1}{2}\dot{q}^T D(q)\dot{q} \tag{7}$$

by developing the previous formula, we obtain :

$$K = \frac{1}{2}\sum_{m,n} d_{m,n}(q)\dot{q}_m\dot{q}_n \tag{8}$$

where $d_{m,n}(q)$ is the m,n^{th} element of the matrix $D(q)$.

The gravity's torque is given by :

$$Q_j = \sum_i g m_i \frac{\partial G_{zi}^0}{\partial q_j}. \tag{9}$$

- G^0_{zi} is the z coordinate of the CoG of the i^{th} segment's computed in the base frame (R_0),
- g is the gravity acceleration.

Vector T (defined in (2)) is composed of the external forces' torque. For the robot presented here, there is no consideration of external forces, so the T vector only describes the motorized torques. Joints 1, 4, 7 and 8 are motorized, so the vector T is given by those four parameters. T_1 and T_4 are computed from the torques of motors 1 and 2 while T_7 and T_8 are deduced from motors 3 and 4.

The Euler-Lagrange equations can be written as :

$$\sum_m d_{jm}(q)\ddot{q}_m + \sum_{n,m} c_{nmj}(q)\dot{q}_n\dot{q}_m = Q_j + T_j \tag{10}$$

$$c_{nmj} = \frac{1}{2}[\frac{\partial d_{jm}}{\partial q_n} + \frac{\partial d_{jn}}{\partial q_m} - \frac{\partial d_{nm}}{\partial q_j}] \tag{11}$$

which is classically written as :

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} = Q + T \tag{12}$$

where $D(q)$ represents the matrix of inertia and $C(q,\dot{q})$ the centrifuge-coriolis matrix where X_{jm}, the jm^{th} element of this matrix, is defined as :

$$X_{jm} = \sum_n c_{nmj}\dot{q}_n.$$

Finally, the J_{vi} and J_{wi} matrix considered in (4) and (5) have to be computed.

2.2 J_{vi} and J_{wi} matrix formulation

The matrix which links articular speed and general speed of a segment is computed from the linear and angular speeds formulas. The goal is to find a matrix for each segment. They are composed of 8 vectors (one for each joint of the model).

The computation consists in formulating in the base frame, the speed ($V_{P_i}(j-1,j)^{R_0}$) of a point P_i given by a motion of the joint q_j attached to the frame j according to the frame $j-1$. Those parameters can be deduced from the law of composition speeds and the Denavitt Hartenberg (DH) formalism used for the geometric model [3]. Indeed, the general formulation is simplified by the geometric model. Only one degree of freedom (DoF) links two frames using the DH model and this DoF is a revolute or a prismatic joint. Moreover, the Z axis is always the rotation or translation axis, so the angular and linear speeds are given by four cases :

- The angular speed of a point for a revolute joint :

$$w_P(j-1,j)^{R_0} = R_{0,j}\begin{bmatrix}0\\0\\1\end{bmatrix}\dot{q}_j. \tag{13}$$

- The linear speed of a point for a revolute joint :

$$\begin{aligned} v_P(j-1,j)^{R_0} &= V_{O_j}^{R_{j-1}} + V_P^{R_j} + w_j \wedge O_j P^{R_j} \\ &= \dot{q}_j R_{0,j} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \wedge R_{0,j} P_j. \end{aligned} \tag{14}$$

- The angular speed of a point for a prismatic joint :

$$w_P(j-1,j) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \tag{15}$$

- The linear speed of a point for a prismatic joint :

$$v_P(j-1,j)^{R_0} = R_{0,j} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \dot{q}_j \tag{16}$$

where P_j is the P point's coordinates in R_j.
Thus, the matrix of a segment i is formulated by computing speeds for each joints as :

$$\begin{bmatrix} v_i \\ w_i \end{bmatrix} = J(q)\dot{q} = [J_{1,i}(q), J_{2,i}(q), \ldots J_{8,i}(q)]\,\dot{q} \tag{17}$$

where $J_{j,i}(q)$ is a vector which links the speed of the i^{th} segment according to the j^{th} joint. The first segment is not affected by the motion of joints 7 and 8 while the second is not affected by joint 8, therefore $J_{7,1}(q)$, $J_{8,1}(q)$ and $J_{8,2}(q)$ are represented by a null vector.

3 ZMP computation

Previous theoretical works and experiments have proved the ZMP efficiency [4]. It consists in keeping the point on the ground at which the moment generated by the reaction forces has no component around x and y axis ([5] and [6]) in the support polygon of the robot. When the ZMP is at the border of the support polygon the robot is teetering. Unlike the ground projection of the center of gravity, it takes into account the robot's inertia.

The purpose of the following is to defined the coordinates of this point in any frame of the model according to the configuration of the robot. The definition can be implemented into the Newton equations to obtain those coordinates. In any point of the model : $M_0 = M_z + OZ \wedge R$ (M_0 and M_z define respectively the moment generated by the reaction force R at the points 0 and z).
According to the previous definition, there is no moment generated by reaction forces at the Zero Moment Point. Consequently, if Z defines the ZMP coordinates $M_0 = OZ \wedge R$. This formulation can be implemented into the Newton equations as :

$$\delta_0 = M_0 + \vec{OG} \wedge \vec{P} + \vec{OG} \wedge \vec{F_i} \tag{18}$$

where P is the gravity force, G is the robot's center of gravity and F_i is the inertial force (the first Newton's law gives $F_i = -m\ddot{G}$). According to the ZMP definition, the equation (18) can be formulated as :

$$\delta_0 = \vec{OZ} \wedge \vec{R} + \vec{OG} \wedge \vec{P} + \vec{OG} \wedge \vec{F_i} \tag{19}$$

$$\begin{cases} \delta_{0x} = Z_y R_z + G_y P_z - G_z P_y - G_z Fi_y \\ \delta_{0y} = -Z_x R_z + G_z P_x - G_x P_z \end{cases} \tag{20}$$

$$\begin{cases} Z_y = \frac{\delta_{0x} - G_y P_z + G_z P_y + G_z Fi_y}{R_z} \\ Z_x = \frac{-\delta_{0y} + G_z P_x - G_x P_z}{R_z}. \end{cases} \tag{21}$$

Also, it is possible to compute the position of the ZMP as a function of q (δ_0 depends on the matrix $D(q)$).

Assuming the ground knowledge, the ZMP computation gives a criterion to determinate the stability of the platform.

.1 Mechanical constants

.1.1 Weight

- Mass :
  - Body 1 : 6.356067 Kg
  - Body 2 : 0.897066 Kg
  - Body 3 : 1.06215 Kg

- Density :
  - Body 1 : 2080.745009 $Kg.m^{-3}$
  - Body 2 : 1739.217853 $Kg.m^{-3}$
  - Body 3 : 2423.47357 $Kg.m^{-3}$

.1.2 Dimensions

- Length :
  - Body 1 : 0.32 m
  - Body 2 : 0.23 m
  - Body 3 : 0.226 m

- Height : 0.60 m

- Width : 0.37 m

- Volume :
  - Body 1 : 0.003055 m^3
  - Body 2 : 0.000516 m^3
  - Body 3 : 0.00044 m^3

- Surface area :
  - Body 1 : 1.715140 m^2
  - Body 2 : 0.362876 m^2
  - Body 3 : 0.30692 m^2

.1.3 Inertia, Center of Mass

- Inertia Matrix ($Kg.m^2$) :
  - Body 1 (defined in the frame R_6) :

$$\begin{pmatrix} 0.128728 & -0.033872 & 0.002259 \\ -0.033872 & 0.363749 & 0.001158 \\ 0.002259 & 0.001158 & 0.283586 \end{pmatrix}$$

  - Body 2 (defined in the frame R_7) :

$$\begin{pmatrix} 0.016485 & -0.000107 & 0.000002 \\ -0.000107 & 0.003941 & 0.001773 \\ 0.000002 & 0.001773 & 0.012863 \end{pmatrix}$$

  - Body 3 (defined in the frame R_8) :

$$\begin{pmatrix} 0.05229 & 0 & -0.00002 \\ 0 & 0.02964 & 0 \\ -0.00002 & 0 & 0.02523 \end{pmatrix}$$

- Center of Mass :
  - Body 1 (in the frame R_6) :

$$\begin{pmatrix} 0.160894 \\ 0.034997 \\ -0.002784 \end{pmatrix}$$

- Body 2 (in the frame R_7) :

$$\begin{pmatrix} -0.000723 \\ -0.104083 \\ 0.014127 \end{pmatrix}$$

- Body 3 (in the frame R_8) :

$$\begin{pmatrix} 0.00009 \\ -0.00001 \\ 0.15268 \end{pmatrix}$$

References

[1] J. J. Craig. *Introduction to Robotics Mechanics and control*. Silma, 2 edition, 1989.

[2] W. Khalil and E. Dombre. *Modeling, Identification and Control of Robots*. Kogan Page Science, 2 edition, 2004.

[3] L. Hardouin J. L. Paillat, P. Lucidarme. Variable geometry tracked vehicle, description, model and behavior. Conference Mecatronics, May 2008.

[4] M. Vukobratovic and B. Borovac. Zero-momment point - thirty five years of its life. *Internationnal journal of humanoid Robotics*, 1(1):157–173, 2004.

[5] Y. Youm J. Kim, W. K. Chung and B. H. Lee. Real time zmp compensation method using null motion for mobile manipulators. Proceedings of the 2002 IEEE Internationnal Conference on Robotics ans Automation, May 2002.

[6] K. Kaneko K. Fujiwara K. Harada K. Yokoi S. Kajita, F. Kanehiro and H. Hirukawa. Biped walking pattern generation by using preview control of zero moment point. Proceedings of the 2002 IEEE Internationnal Conference on Robotics ans Automation, September 2003.